Halo profiles from weakly nonlinear initial conditions

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Abstract. Using weakly nonlinear conditional PDF for the density field around an overdense region we find that the expected density contrast around a peak is always smaller while its rms fluctuation larger than in the linear case. We apply these results to the spherical model of collapse as developed by Hoffman & Shaham (1985) and find that in the case of open Universe the effect of weakly nonlinear interactions is to decrease the scale from which a peak gathers mass and the mass itself as well as to steepen the final profile of the virialized protoobject.

1. Introduction

The purpose of this work was to find a generalization of the spherical infall model as developed by Hoffman & Shaham (1985, hereafter HS) to the case of density peaks collapsing in the weakly nonlinear background. One of the key assumptions underlying the calculations of HS was that the matter influenced by the peak collapses onto it undisturbed by the background. This is equivalent to the statement that the peak identified with some resolution (smoothing) scale collapses while the surrounding density field is still linear i.e. its rms fluctuation at this scale is much smaller than unity. The example of the Virgo supercluster (which has not yet collapsed) shows that this is not always the case: the rms fluctuation at the scale of the supercluster is well in the weakly nonlinear regime.

In this generalization we hope to account properly for the weakly nonlinear transition between the linear and strongly nonlinear phase of the evolution of the perturbation which lacked in the approach of HS. It involves constructing the weakly nonlinear probability distribution function (PDF) of density around an overdense region. The mean density obtained from this weakly nonlinear PDF is then taken as the initial condition for spherical collapse.

The reliability of this approach rests on the assumption that the influence of the neighbouring fluctuations can be restricted to the weakly nonlinear phase with its only outcome in the form of a changed 'initial' density profile which then evolves independently of surroundings, according to the spherical model.

2. Conditional probability distribution around a peak

We consider density contrast field which initially has Gaussian distribution with zero mean. The field measured at a given point is denoted by δ while the one measured at the distance r from the first point is called γ. We smooth the

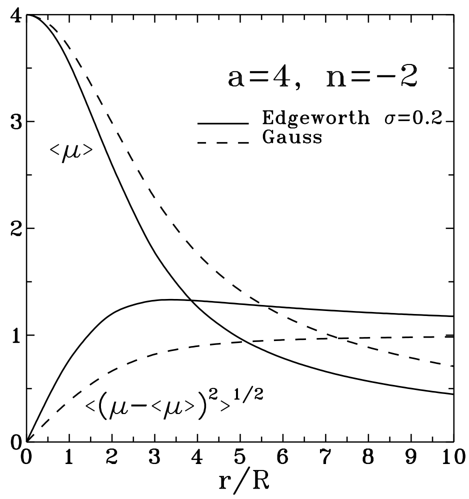

Figure 1. The expected normalized density contrast $\langle\mu\rangle$ and the dispersion $\langle(\mu - \langle\mu\rangle)^2\rangle^{1/2}$ for the Edgeworth and Gaussian conditional distributions as a function of the distance from the peak.

fields with Gaussian filters of the same scale R so their variances are equal $\langle\delta^2\rangle = \langle\gamma^2\rangle = \sigma^2$.

We generalize the joint probability distribution function for density contrast field in the Gaussian case to the case when the density fields are weakly nonlinear and therefore departing from Gaussianity. If the rms fluctuations of the fields are small (below unity) the fields can be expanded around their linear values δ_1 and γ_1. The resulting two-point PDF takes the form of the bivariate Edgeworth series (Łokas 1997).

Let us now suppose that the value of one variable is known: $\gamma = a\sigma$, i.e. at $r = 0$ we have a region of a standard deviations. The conditional probability distribution of the normalized density $\mu = \delta/\sigma$ at distance r from this region is well characterized by its lowest order moments with respect to the mean value, $\langle\mu\rangle$. In the Gaussian case we have $\langle\mu\rangle_G = \varrho a$ and $\langle(\mu - \langle\mu\rangle)^2\rangle_G = 1 - \varrho^2$, while in the weakly nonlinear approximation we get (Łokas 1997)

$$\langle\mu\rangle = \varrho\, a + \frac{\sigma}{2}(a^2 - 1)(S_{12} - \varrho S_3) \tag{1}$$

$$\langle(\mu - \langle\mu\rangle)^2\rangle = 1 - \varrho^2 + \sigma\, a\, (S_{12} - 2\varrho S_{12} + \varrho^2 S_3) \tag{2}$$

where $\varrho = \xi_R(r)/\sigma^2$ is the correlation coefficient and S_3 and S_{12} are the one-point and two-point skewness parameters respectively.

It turns out that for Gaussian smoothing, scale-free power spectra $P(k) = Ck^n$ with indices $n = -2, -1.5, -1$ we have $S_{12} - \varrho S_3 < 0$. This proves that according to equation (1) for $a > 1$ (peaks) the correction to the mean normalized density contrast with respect to the Gaussian value is always negative.

Figure 1 shows that the effect of weakly nonlinear interactions is to decrease the expected density around an overdense region. In the case of the variance (Figure 1) the weakly nonlinear corrections work in the opposite direction: their effect is to increase the value of the variance (or dispersion) with respect to the linear case, because for the power spectra considered here we always have $S_{12} - 2\varrho S_{12} + \varrho^2 S_3 > 0$.

It is clear that a local density peak that rises significantly above the noise should gravitationally dominate its surroundings out to some distance. A reasonable measure of the distance, up to which a coherent structure around the peak is expected, is the scale r_{coh} at which $\langle\mu\rangle = \langle(\mu - \langle\mu\rangle)^2\rangle^{1/2}$. Figure 1 shows that the coherence scale in weakly nonlinear regime is decreased with respect to the linear case.

3. Application to spherical collapse

The dynamical evolution of matter at the distance $c_i = r_i/R$ from the peak is determined by the mean cumulative density perturbation within c_i which in weakly nonlinear approximation preserves its slope $c_i^{-(n+3)}$ for large c_i but the height of the peak instead of the linear value a can be approximated by $a_{eff} = a[1 - \sigma s(a^2-1)/(2a)]$, where s is of the order of unity. If we now assume that the matter within distance c_i from the peak collapses undisturbed onto the peak, the spherical model can be applied. Following the calculations of HS with the initial conditions settled by the Edgeworth approximation instead of the Gaussian one we find the final density profile to be given by

$$\rho(c) \propto \frac{[(c_0/c_i)^{n+3} - 1]^4}{(n+4)(c_0/c_i)^{n+3} - 1}, \qquad c_i/c \propto (c_0/c_i)^{n+3} - 1 \tag{3}$$

which has the same form as that of HS (so that the limiting cases $\rho(c) \propto (c_0/c)^{3(n+3)/(n+4)}$ for $c_i \ll c_0$ and $\rho(c) \propto (c_0/c)^4$ for $c_i \leq c_0$ are preserved) but the scale from which the peak gathers mass (the distance to the shell of zero energy) is decreased with respect to the linear case

$$c_0 = c_{0,G}\left[1 - \frac{\sigma s(a^2-1)}{2a(n+3)}\right]. \tag{4}$$

The value of c_0 determines the slope of the profile (3), the shorter c_0 the steeper is the density profile. The value of c_0 is defined so that in flat Universe it is infinite therefore in this case the weakly nonlinear corrections do not affect the final profile. In the open Universe, however, the correction to c_0 is significant and leads to the steepening of the final profile and decreasing the mass bound to the peak.

References

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